



Progression In

Written Methods

Of Calculation
(Updated October 2022)

MATHS MASTERY APPROACH at ESH WINNING PRIMARY SCHOOL

Since September 2018, we have embraced the Teaching for Mastery Approach. This is a long-term approach which will allow children to have a deeper understanding of maths and allow them to use these skills in other subjects, showing that skills learnt in maths can be transferable.

For Reception to Year 6, the White Rose Long Term Plans will be used and the White Rose Small Steps Approach will be used to ensure an accurate teaching sequence. Through taking small steps in our mathematical journey and working hard, we believe that every child can succeed at maths.

In essence, the Maths Mastery Approach means:

- The majority of the class will be taught together, learning the same maths at the same time
- A carefully planned learning journey of small steps will be taken in order to ensure that all children master the concepts before moving on so that no child is left behind
- If a pupil requires extra support, this is identified quickly, and where possible, same day intervention takes place
- Children learn concepts using the **CONCRETE – PICTORIAL – ABSTRACT** Approach.

PROGRESSION IN CALCULATION

From Foundation Stage to Year 6

The aim for written calculations is different from the aim for mental calculations. With mental work, the aim is to teach children a repertoire of strategies from which to select. With written calculations the ultimate aim is proficiency in a compact method for each operation.

Mental calculation

Strategies for mental calculation are introduced from Y1 to Y3 and developed further in Y4, Y5 and Y6. All children, apart from those with significant special educational needs, should be introduced to the full range of mental calculation strategies when they have the necessary pre-requisite skills. Children with significant special needs should learn a narrow range of strategies which are generally applicable.

Weekly Timetables Olympics are carried out from Y2 (with children who are achieving Greater Depth in Year 1 also accessing it). Children progress through the different levels. Recognition and Rewards are given for achieving particular levels.

Times Tables Rock Stars is used on a regular basis for children to practise their fluency in times tables knowledge. The Times Tables Rock Stars Sessions Bolt-On is used as part of homelearning from Years 2 – 6.

All children from Y1 – Y6 have a mental maths activity recorded in books at least three times a week. Mental Maths Activities which are directly linked to the concept being taught are recorded in the front of children's books, whereas those not directly linked or Times Tables Practise are recorded in the back of children's books.

Mathstastic is carried out weekly and children progress through their Year Group Maths Curriculum. Those children with special educational needs will progress through previous Year Groups Maths Curriculum and will move onto the next level as soon as they are ready. Those children who complete their Year Group Mathstastic will **NOT** move onto the next Year Group Mathstastic but will carry out a Problem Solving and Reasoning Question.

Written calculations

Building on the mental strategies they have used so that they can understand the process, children first need to be taught to record their methods in an expanded form. When ready, they are taught how to refine the recording to make it more compact. The methods and layouts to be taught for each operation are detailed in this document.

Problem Solving and Reasoning

At every step, throughout their maths journey, children will be exposed to Problem Solving and Reasoning questions, using the four operations. This will give children the opportunity to apply their maths knowledge and explain or show, using appropriate mathematical language, how they have solved the problem.

Challenges to teachers

- Ensuring that recall skills are established first so children can concentrate on a written method without reverting to first principles.
- Making sure that, once written methods are introduced, children continue to look out for and recognise the special cases that can be done mentally.
- Catering for children who progress at different rates - some may grasp a compact method of calculation while others may never do so without considerable help.
- Catering for children who can carry out some standard methods successfully, e.g. for +, but not - .
- Recognising that children tend to forget a standard method if they have no understanding of what they are doing.

Often the compactness of a vertical method conceals how mathematical principles are applied, e.g. children may use place value when working mentally, but be confused in written work because they do not understand how place value relates to exchanging. There can be long-lasting problems for those taught compact, vertical methods before they understand what they are doing.

Simply correcting children's errors may help in the short-term, but not permanently. They need to understand why a particular method works rather than simply following a set of rules. They can then fall back to a simpler method if uncertain or to check their answer.

NOTE: In the following guidance, suggestions are given as to when written methods and particular layouts should be introduced. However, the most important thing to consider rather than children's age, is whether they have the necessary pre-requisite skills.

Progression towards a written method for addition

Before the introduction of formal written methods for addition, children should be able to:

1. Recall all addition pairs to $9 + 9$ and complements in 10;
2. Add mentally a series of one-digit numbers, such as $5 + 8 + 4$;
3. Add multiples of 10 (such as $60 + 70$) or of 100 (such as $600 + 700$) using the related addition fact, $6 + 7$, and their knowledge of place value;
4. Partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.

EARLY STAGES (EYFS)

Children will engage in a wide variety of songs, rhymes, games and activities.

They will begin to relate addition to combining two groups of objects, first by counting all and then by counting on from the largest number.

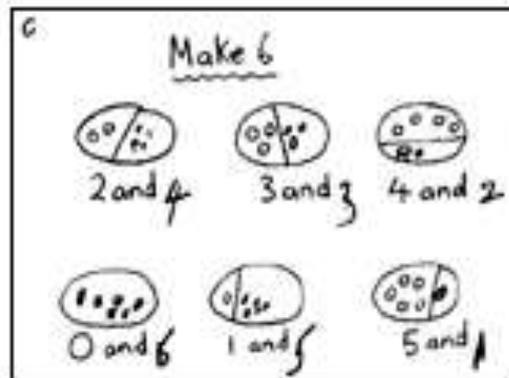
They will find one more than a given number.

In practical activities and through discussion they will begin to use the vocabulary involved in addition.



'You have five apples and I have three apples. How many apples are there altogether?'

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.



YEAR 1

Statutory requirements

Pupils should be taught to:

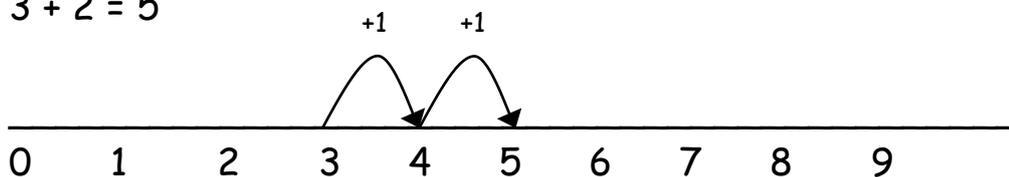
- read, write and interpret mathematical statements involving addition (+) and equals (=) signs
- represent and use number bonds within 20
- add one-digit and two-digit numbers to 20, including zero
- solve one-step problems that involve addition, using concrete objects and pictorial representations, and missing number problems such as: $7 = \square + 5$.

Children use number tracks, number lines and practical resources, to support calculation.



They count on from the largest number.

$$3 + 2 = 5$$



YEAR 2

Statutory requirements

Pupils should be taught to:

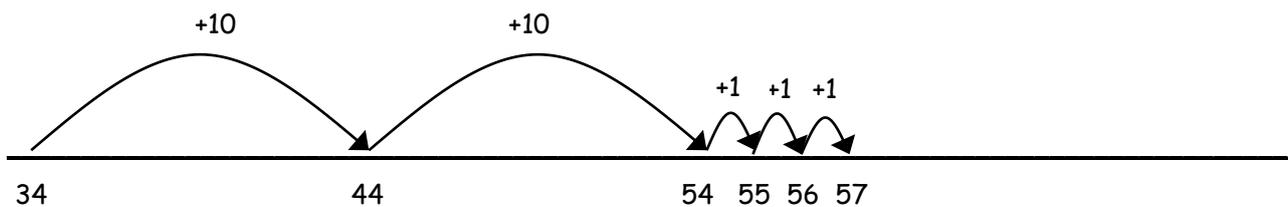
add numbers using concrete objects, pictorial representations, and mentally, including:

- a two-digit number and ones
- a two-digit number and tens
- two two-digit numbers
- adding three one-digit numbers

Children will begin to use 'empty number lines' themselves starting with the larger number and counting on.

First counting on in tens and ones.

$$34 + 23 = 57$$



Children then progress to more efficient methods by adding units in one jump and then tens in one jump.

When adding two-digit numbers the children can use partitioning which can help with mental calculations too.

$$26 + 43 =$$

$$20 + 40 = 60$$

$$6 + 3 = 9$$

$$60 + 9 = 69$$

In Year 2, children are also introduced to the column addition method, which will also support their understanding of place value.

$$26$$

$$\underline{43} +$$

$$\underline{69}$$

YEAR 3

Statutory requirements

Pupils should be taught to:

- add numbers with up to three digits, using formal written methods of columnar addition

Build up starting with 2 digits + 2 digits

Introducing the compact layout

- no carrying (demonstration stage only), e.g. $54 + 35$, $326 + 271$
- carrying from units to tens, e.g. $47 + 26$, $368 + 423$
- carrying from tens to hundreds, e.g. $368 + 481$
- carrying from units to tens and tens to hundreds, e.g. $47 + 76$, $368 + 478$
- a mixture of 'carries'

E.g. demonstrate the method children currently use alongside the new method. Children should then practise the new method. ($368 + 478$)

$$\begin{array}{r} \text{H T O} \\ 368 \\ + 478 \\ \hline 846 \\ 11 \end{array}$$

When the compact layout is introduced, it is helpful to use the place value headings of HTO.

When the compact layout is introduced, the language of place value should continue to be used but when children are confident, they will use 'digit-speak', e.g. for the addition of two three-digit numbers above, they are likely to say:

- 8 add 8 is sixteen; 6 in the answer and carry 1
- 6 add 7 is 13 plus the carry 1 is 14; 4 in the answer and carry 1
- 3 add 4 is 7 plus the carry 1 is 8
- Answer is 846

This is a form of shorthand that speeds up the process of addition. If children consistently carry out a range of calculations correctly, it is likely that they understand the process. However, teachers should occasionally check their understanding by asking children to explain what exactly they mean at each stage of the calculation, e.g. what does that 'carry 1' really mean?

YEAR 4

Statutory requirements

Pupils should be taught to:

- add numbers with up to 4 digits using the formal written methods of columnar addition where appropriate

See the column method shown for Y3

Children in Y4 will meet some fairly simple additions of £ and p.

Some children may be able to use the compact column method, but those who have not learnt how to use this method should change pounds to pence and add using a compact method.

Ensure that children are aware that the decimal points should line up especially when using mixed amounts.
Eg £3.59 + 78p =

$$\begin{array}{r} 3.59 \\ 0.78 + \\ \hline 4.37 \\ 11 \end{array}$$

YEAR 5

Statutory requirements

Pupils should be taught to:

- add whole numbers with more than 4 digits, including using formal written methods (columnar addition)

Continue column addition method taught in earlier year groups.

These should be extended through addition of money and measures.

YEAR 6

There are no formal statutory requirements for addition in Y6. They are expected to continue with the formal written methods and apply this to problem solving.

Progression towards a written method for subtraction

EARLY STAGES (EYFS)

Children will engage in a variety of counting songs and rhymes and practical activities.

In practical activities and through discussion they will begin to use the vocabulary associated with subtraction.

They will find one less than a given number.

They will begin to relate subtraction to 'taking away' using objects to count 'how many are left' after some have been taken away.

$$6 - 2 = 4$$



Take two apples away. How many are left?

Children will begin to count back from a given number.

YEAR 1

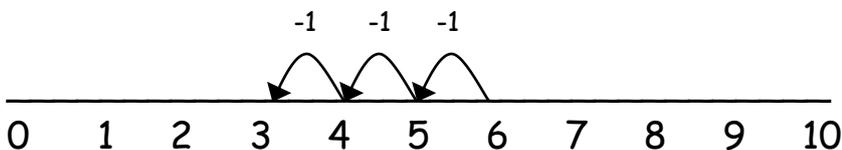
Statutory requirements

Pupils should be taught to:

- read, write and interpret mathematical statements involving subtraction (−) and equals (=) signs
- subtract one-digit and two-digit numbers to 20, including zero

The number line should be used as the children will be using this for addition too.

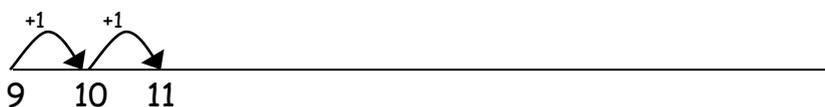
$$6 - 3 = 3$$



The number line should also be used to show that $6 - 3$ means the 'difference between 6 and 3' or 'the difference between 3 and 6' and how many jumps they are apart.

Children should also be taught the counting on method, using a number line; this will establish the relationship between the operations of addition and subtraction.

$$11 - 9 = 2$$



YEAR 2

Statutory requirements

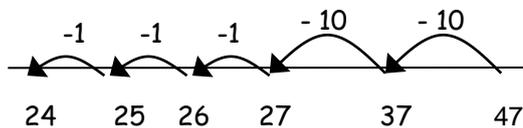
Pupils should be taught to:

- subtract numbers using concrete objects, pictorial representations, and mentally, including:
 - a two-digit number and ones
 - a two-digit number and tens
 - two two-digit numbers

Children can continue to use a number line counting back and counting on when finding small differences.

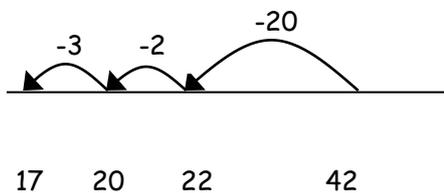
First counting back in ones then progressing to tens and ones and then beyond.

$$47 - 23 = 24$$



Bridging through ten can help children become more efficient.

$$42 - 25 = 17$$



Counting on to find a small difference

$$52 - 49 = 3$$



Children should be taught to use the column subtraction method without any exchanges. This supports place value and prepares for formal written methods with larger numbers.

$$56 - 23 = 33$$

$$\begin{array}{r} \text{T O} \\ 56 - \\ \underline{23} \\ 33 \end{array}$$

When the formal layout is introduced, it is helpful to use the place value headings of TO.

YEAR 3

Statutory requirements

Pupils should be taught to:

- subtract numbers with up to three digits, using formal written methods of columnar subtraction

Children should be confident in carrying out subtraction by counting back or on, using a number line, before being introduced to the formal method of decomposition. Some children find decomposition difficult.

Decomposition

Before the introduction of the formal written method for subtraction (decomposition), children should be able to:

- Recall all subtraction facts to 20;
- Subtract multiples of 10 (such as $160 - 70$) using the related subtraction fact, $16 - 7$, and their knowledge of place value;
- Partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into $70 + 4$ or $60 + 14$).

Teach the decomposition method in this order:

1. Exchange from tens to ones, for example;
TO – TO; $71 - 46$, HTO – TO; $173 - 38$ HTO – HTO; $774 - 248$,
2. Exchange from hundreds to tens, for example;
HTO – HTO; $553 - 272$
3. Exchange from tens to ones and from hundreds to tens, for example;
HTO – HTO; $635 - 278$

$$71 - 46 =$$

This should be formally recorded.

$$\begin{array}{r} 6 \ 1 \\ \cancel{7} \ 1 \\ - 4 \ 6 \\ \hline 2 \ 5 \end{array}$$

YEAR 4

Statutory requirements

Pupils should be taught to:

- subtract numbers with up to 4 digits using the formal written methods of columnar subtraction where appropriate

Continue, or begin, with the formal written method of decomposition as set out in Y3.

Extending to;

- Subtract numbers with up to four digits, including numbers with different numbers of digits
- Subtraction with numbers involving zeros, for example; $5001 - 2345$
- Subtract decimals, money and measures

$874 - 523$ becomes	$932 - 457$ becomes
$\begin{array}{r} 874 \\ - 523 \\ \hline 351 \end{array}$	$\begin{array}{r} 8 \quad 12 \quad 1 \\ 932 \\ - 457 \\ \hline 475 \end{array}$
Answer: 351	Answer: 475

YEAR 5

Statutory requirements

Pupils should be taught to:

- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)

Follow column subtraction (decomposition) method taught in earlier year groups.

YEAR 6

There are no formal statutory requirements for subtraction in Y6. They are expected to continue with the formal written methods and apply this to problem solving.

Progression towards a written method for multiplication

EARLY STAGES (EYFS)

Children will engage in a wide variety of songs, rhymes, games and activities.

In practical activities and through discussion they will begin to solve problems involving doubling.



'Three apples for you and three apples for me. How many apples altogether?'

YEAR 1

Statutory requirements

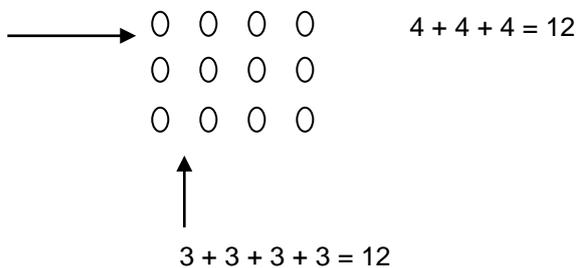
Pupils should be taught to:

- solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

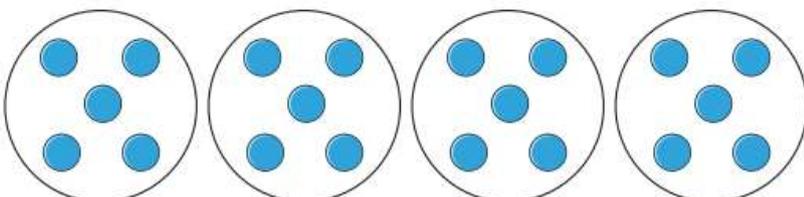
Pictorial recording as children develop understanding of repeated addition, e.g. counting in sets of two, e.g. sets of two pence coins, five, e.g. tally bundles, tens, e.g. sets of 10 pennies.



The use of arrays to illustrate repeated addition.



Making equal groups



YEAR 2

Statutory requirements

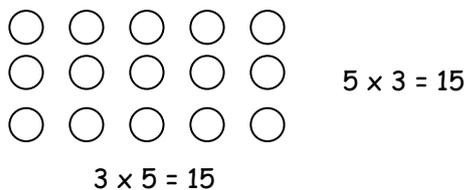
Pupils should be taught to:

- recall and use multiplication facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (\times) and equals ($=$) signs
- show that multiplication of two numbers can be done in any order (commutative)

Children should know that 3×5 has the same answer as 5×3 .

Children should be able to model a multiplication calculation using an array.

The use of arrays to illustrate repeated addition.



Making equal groups with the addition of using the correct vocabulary and the operation.



lots of =

multiplied by =

\times =

Creating multiplication sentences with repeated addition and the multiplication operation



There are equal groups with
in each group.

$$\square + \square + \square = 18$$

$$\square \times \square = 18$$

Year 3 to Year 6

Before the introduction of formal written methods for multiplication, children should be able to:

- Recall multiplication facts for the tables used;
- Partition numbers into multiples of one hundred, ten and one;
- Work out products such as 70×5 , 70×50 , 700×5 or 700×50 using the related fact 7×5 and their knowledge of place value;
- Add two or more single-digit numbers mentally;
- Add multiples of 10 (such as $60 + 70$) or of 100 (such as $600 + 700$) using the related addition fact, $6 + 7$, and their knowledge of place value;
- Add combinations of whole numbers using the column method.

YEAR 3

Statutory requirements

Pupils should be taught to:

- recall and use multiplication facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods

Children will learn short multiplication methods of a TO x O with no exchanges

	T	O
	4	3
x		2
	<u>8</u>	<u>6</u>

Once children are confident with this method, they will be introduced to multiplying a TO x O with an exchange

		T	O
		3	5
x			6
	<u>2</u>	<u>1</u>	<u>0</u>
		3	

YEAR 4

Statutory requirements

Pupils should be taught to:

- multiply two-digit and three-digit numbers by a one-digit number using formal written layout

Children should progress to using the formal written layout for multiplication problems.

Multiplying a TO x O with an exchange

	H	T	O
		4	2
x			8
	3	3	6
		1	

Multiplying an HTO x O with an exchange

	H	T	O
	2	1	5
x			3
	6	4	5
		1	

This can also be extended to include money: £ and p x O, for example, £3.64 x 8

YEAR 5

Statutory requirements

Pupils should be taught to:

- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply whole numbers and those involving decimals by 10, 100 and 1000

Short Multiplication

Multiplying an ThHTO x O without an exchange

	2	1	0	2
×				4
	8	4	0	8

Multiplying an ThHTO x O with an exchange

	2	2	1	4
×				4
	8	8	5	6
			1	

Long Multiplication

Multiplying a TO x TO

		4	3
×		1	3
	1	2	9
	4	3	0
	5	5	9

Multiplying an HTO x TO

		2	3	1	
x			1	3	
		6	9	3	
	2	3	1	0	
	3	0	0	3	
	1	1			

(231 × 3)

(231 × 10)

YEAR 6

Statutory requirements

Pupils should be taught to:

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication

Children should apply the formal written methods taught in earlier year groups to more complex numbers, including decimals and measures.

Progression towards a written method for division

EARLY STAGES (EYFS)

Children will engage in a wide variety of songs, rhymes, games and activities.

In practical activities and through discussion they will begin to solve problems involving halving and sharing.

Share the apples between two people.



'Half of the apples for you and half of the apples for me.'

YEAR 1

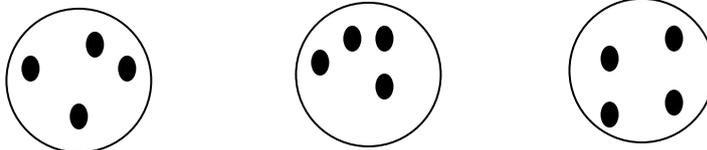
Statutory requirements

Pupils should be taught to:

- solve one-step problems involving division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

- Use **sharing** to answer division questions such as:

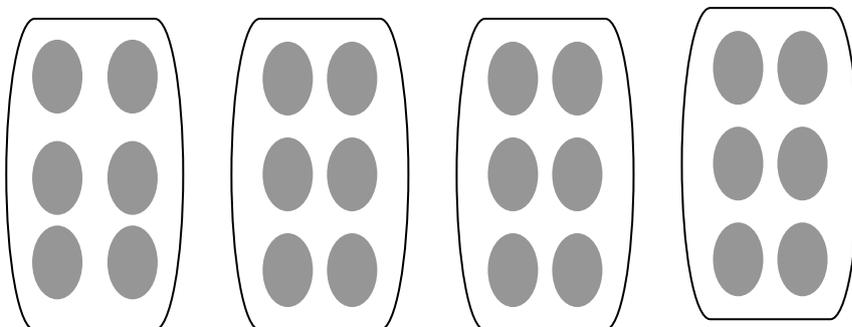
Share this bag of 12 counters into 3 pots.



- Experience **division as grouping**, such as:

24 eggs are packed in boxes of 6. How many boxes are needed?

Take 6 eggs and pack the first box. Continue until there are no eggs left. Then count how many boxes have been used, recording pictorially.



YEAR 2

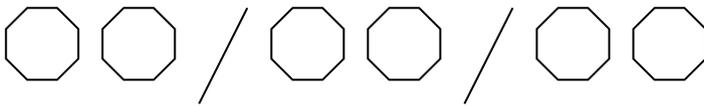
Statutory requirements

Pupils should be taught to:

- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- Calculate mathematical statements for division and write them using the division (\div) and equals ($=$) signs
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot

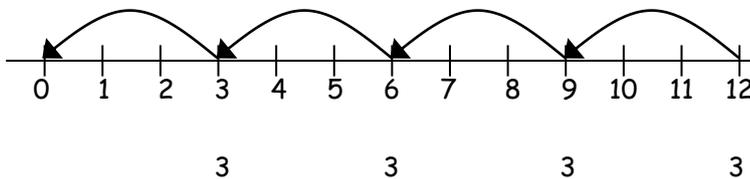
Introduce the number sentence and allow children to use sharing or grouping pictorial representations to solve.

There are 6 sweets, how many people can have 2 sweets each? $6 \div 2 = 3$



Repeated subtraction using a number line or bead bar

$$12 \div 3 = 4$$



Using symbols to stand for unknown numbers to complete equations using inverse operations

$$\square \div 2 = 4$$

$$20 \div \triangle = 4$$

$$\square \div \triangle = 4$$

Experience divisions that give rise to **remainders**, such as, in a sharing context:

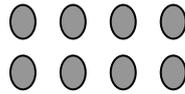
Three friends share 17 marbles equally. How many marbles does each friend get? How many marbles are left over?



$$17 \div 3 = 5 \text{ r } 2 \text{ (5 with 2 left over)}$$

Relate division to multiplication using arrays.

How many sets of four in 8? →



$$8 \div 4 = 2$$



How many sets of two in 8?

$$8 \div 2 = 4$$

Year 3 to Year 6

To progress towards short division, children need to be able to:

- Understand and use the vocabulary of division;
- Partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- Recall multiplication and division facts for the tables used;
- Recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- Know how to find a remainder working mentally, e.g. find the remainder when 48 is divided by 5;
- Understand and use multiplication and division as inverse operations.

YEAR 3

Statutory requirements

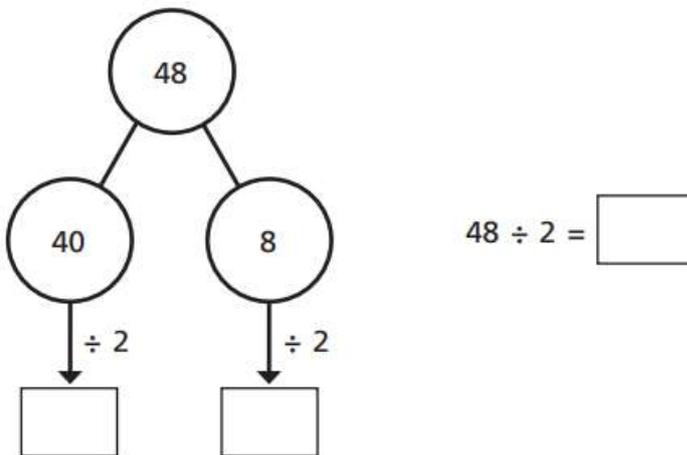
Pupils should be taught to:

- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for division using mental and progressing to formal written methods

Expanded layout for short division

Model this to support understanding of the place value within the compact method but children are not expected to record this method. Record in compact form (short division)

Part-Part Whole Model



Children will progress to short division of a $TO \div O$, using the 'bus stop' method with no remainders

$$\begin{array}{r} 31 \\ 3 \overline{) 93} \end{array}$$

Once children are confident with this method, they will progress to short division with remainders of a $TO \div O$ but no exchanges

$$\begin{array}{r} 22 \text{ r } 1 \\ 3 \overline{) 67} \end{array}$$

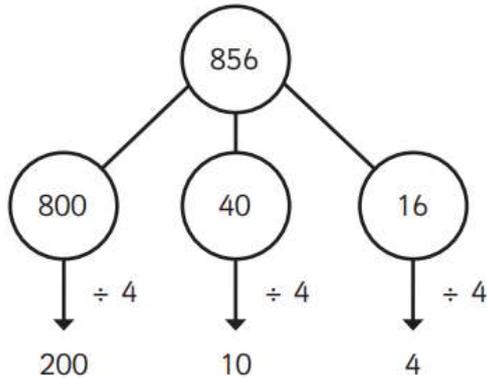
YEAR 4

Statutory requirements

- There are no formal statutory requirements linked to division in Year 4.

Children should practise and consolidate the division work in Year 3.

They will begin to divide an HTO \div O using the **Part-Part-Part Whole Model**



Once children are confident with Part-Part-Part Whole and short division with and without remainders, they will progress to short division with exchanges.

TU \div O, carrying from T to O but no remainder, e.g. $76 \div 4$. When dealing with carrying figures, relate to knowledge of place value.

$$\begin{array}{r} 18 \\ 4 \overline{) 736} \end{array}$$

Once children are confident with short division with exchanges, they will then progress to short division, with exchanges and remainders.

TU \div O with exchanges and remainders, e.g. $96 \div 7$

$$\begin{array}{r} \text{T U} \\ 13 \text{ r } 5 \\ 7 \overline{) 926} \end{array}$$

YEAR 5

Statutory requirements

Pupils should be taught to:

- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.

Continue to develop the short division method taught in previous year groups and then progress to short division methods

ThHTO \div O with exchanges but no remainders

	4	2	6	6
2	8	5	'3	'2

ThHTO \div O with exchanges and remainders

	2	5	3	1	r2
3	7	'5	9	5	

YEAR 6

Statutory requirements

Pupils should be taught to:

- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context

Children will continue to develop the short division methods taught in previous year groups and then progress to:

ThHTO \div TO with exchanges

	0	1	6	7
12	2	² 0	⁸ 0	⁸ 4

Children will then progress to long division with no exchanges or remainders of an HTO \div TO

		2	1
13	2	7	3
	2	6	
		1	3
		1	3
			0

Once children are confident with long division of an HTO \div TO with no exchanges, they will then progress to a long division of an HTO \div TO, with a remainder

		4	9	r5
15	7	4	0	
	6	0		
		1	4	0
		1	3	5
			5	

Challenge children with extended digits, decimal numbers, money and measures including remainders